Techno ACE

Illustrated Answer for Model Question Paper

Mathematics

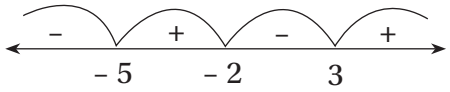
Class : XI going to XII

1. $\sin 15° = \sin(45° - 30°) = \sin 45°\cos 30° - \cos 45°\sin 30°$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Ans. Ⓐ

2. $\frac{(x+2)(x+5)}{(x-3)} < 0$



$\therefore \quad x \in (-\alpha, -5) \cup (-2, 3)$

Ans. Ⓓ

3. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 10 + 15 - 2 = 23$

Ans. Ⓐ

4. For $Z = 0$, Value $= 9 + 36 = 45$

For $Z = 1 + Zi$, Value $= 5 + 8 + 17 = 30$

For $Z = 1 + 3i$, Value $= 10 + 13 + 10 = 33$

For $Z = 1 + 6i$, Value $= 37 + 10 + 1 = 48$

$\therefore \quad Z = 1 + 2i$

alt $|Z|^2 + |Z-3|^2 + |Z-6i|^2$

$= x^2 + y^2 + (x-3)^2 + y^2 + x^2 + (y-6)^2$

$= 3x^2 + 3y^2 - 6x - 12y + 45$

$= 3[(x-1)^2 + (y-2)^2] + 30 \geq 30$

Equality holds when $x = 1$, $y = 2$

$\therefore z = x + iy = 1 + 2i$

Ans. Ⓑ

5. $\sin^2 5° + \sin^2 10° + \sin^2 15° + \ldots + \sin^2 85°$

$= (\sin^2 5° + \cos^2 5°) + (\sin^2 10° + \cos^2 10°) + \ldots + (\sin^2 40° + \cos^2 40°) + \sin^2 45°$

$= 1 + 1 + \ldots + 1 + \frac{1}{2} = 8 + \frac{1}{2} = \frac{17}{2}$

Ans. Ⓓ

6. $f(x) = \text{sgn}(x^2 - 2x + 4) = 1;\ x^2 - 2x + 4 > 0$

$= 0;\ x^2 - 2x + 4 = 0$

$= -1;\ x^2 - 2x + 4 < 0$

$x^2 - 2x + 4 = (x-1)^2 + 3 \geq 3$

$\therefore$ Range = {1}

Ans. Ⓐ

7. $\dfrac{\sin 9^\circ \sin 21^\circ \sin 39^\circ \sin 51^\circ \sin 69^\circ \sin 81^\circ}{\sin 54^\circ}$

$= \dfrac{\sin 9^\circ \cos 9^\circ \sin 21^\circ \cos 21^\circ \sin 39^\circ \cos 39^\circ}{\sin 54^\circ}$

$= \dfrac{1}{8} \times \dfrac{\sin 18^\circ \sin(60^\circ - 18^\circ)\sin(60^\circ + 18^\circ)}{\sin 54^\circ}$

$= \dfrac{1}{8} \dfrac{\sin 18^\circ \sin(60^\circ - 18^\circ)\sin(60^\circ + 18^\circ)}{\sin 54^\circ}$

$= \dfrac{1}{8} \dfrac{\sin 18^\circ (\sin^2 60^\circ - \sin^2 18^\circ)}{\sin 54^\circ}$

$= \dfrac{1}{8} \dfrac{\sin 18^\circ (\frac{3}{4} - \sin^2 18^\circ)}{\sin 54^\circ} = \dfrac{1}{32} \dfrac{(3\sin 18^\circ - 4\sin^3 18^\circ)}{\sin 54^\circ}$

$= \dfrac{1}{32} \dfrac{\cancel{\sin 54^\circ}}{\cancel{\sin 54^\circ}} = \dfrac{1}{32}$

Ans. Ⓒ

8. T — — — — — — — — T

$\therefore$ Number of 10 lettered words $= \dfrac{\lfloor 8}{\lfloor 2} = \dfrac{\lfloor 8}{2}$

Ans. Ⓐ

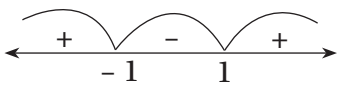
9. No of vowels = 5 where 2I

Taking 5 vowels together and considering as one number of letters 6 where two T.

$\therefore$ Required number $= \dfrac{6!}{2!} \times \dfrac{5!}{2!} = \dfrac{(6!)(5i)}{4}$

Ans. Ⓑ

10. $f(x) = \dfrac{x}{\sqrt{x^2 - 1}}$ $\qquad x^2 - 1 > 0$



$\Rightarrow (x+1)(x-1) > 0$

$\therefore x \in (-\infty, -1) \cup (1, \infty)$

Ans. Ⓐ

11. $\cos 4^\circ \cos 8^\circ \cos 12^\circ \ldots\ldots \cos 88^\circ = 2^{-n}$

$\Rightarrow \dfrac{1}{2\sin 4^\circ} \times 2\sin 4^\circ \cos 4^\circ \cos 8^\circ \cos 12^\circ \ldots \cos 88^\circ = 2^{-n}$

$$\Rightarrow \frac{1}{2^2 \sin 4^\circ} \times 2\sin 8^\circ \cos 8^\circ \cos 12^\circ \ldots \cos 88^\circ = 2^{-n}$$

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$$\Rightarrow \frac{1}{2^{22} \sin 4^\circ} \times \sin 176^\circ = 2^{-n}$$

$$\Rightarrow \frac{1}{2^{22} \cancel{\sin 4^\circ}} \times \cancel{\sin 4^\circ} = 2^{-n}$$

$$\Rightarrow 2^{-22} = 2^{-n} \Rightarrow n = 22$$

Ans. Ⓑ

12. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 6 + 5 - 2 = 9$

Ans. Ⓐ

13. $(x + 2)(x + 3)(x + 8)(x + 12) - 4x^2$

$\Rightarrow (x + 2)(x + 12)(x + 3)(x + 8) - 4x^2 = 0$

$\Rightarrow (x^2 + 14x + 24)(x^2 + 11x + 24) - 4x^2 = 0$

$\Rightarrow (x + \frac{24}{x} + 14)(x + \frac{24}{x} + 11) - 4 = 0$

$\Rightarrow (a + 14)(a + 11) - 4 = 0$ where $a = x + \frac{24}{x}$

$\Rightarrow a^2 + 25a + 154 - 4 = 0$

$\Rightarrow a^2 + 25a + 150 = 0$

$\Rightarrow (a + 15)(a + 10) = 0$

$\therefore a = -15, \quad a = -10$

$\Rightarrow x + \frac{24}{x} = -15, \quad x + \frac{24}{x} = -10$

$\Rightarrow x^2 + 15x + 24 = 0, \quad x^2 + 10x + 24 = 0$

$\Rightarrow x = \frac{-15 \pm \sqrt{225 - 96}}{2}$; $(x + 6)(x + 4) = 0 \Rightarrow x = -6, -4$

$= \frac{-15 \pm \sqrt{129}}{2}$ both negative roots

$\therefore$ Reqd. sum $= -15 - 6 - 4 = -25$

Ans. Ⓒ

14. Total of 50 numbers $= 50 \times 38$

$\therefore$ Total of 48 numbers $= 50 \times 38 - 55 - 45$

$= 5(380 - 11 - 9)$

$= 5 \times 360$

$\therefore$ Reqd. A.M. $= \dfrac{5 \times \cancel{360}^{\overset{15}{30}}}{\cancel{48}_{\cancel{4}_2}} = 37.5$

Ans. Ⓑ

15. $\alpha + \beta = -\dfrac{b}{a}, \alpha\beta = \dfrac{c}{a}, \Delta = b^2 - 4ac$

$(\alpha^2 + \beta^2)^2 = (\alpha + \beta)\,(\alpha^3 + \beta^3)$

$\Rightarrow \cancel{\alpha^4} + \cancel{\beta^4} + 2\alpha^2\beta^2 = \cancel{\alpha^4} + \alpha\beta^3 + \beta\alpha^3 + \cancel{\beta^4}$

$\Rightarrow \alpha\beta^3 + \beta\alpha^3 - 2\alpha^2\beta^2 = 0$

$\Rightarrow \alpha\beta(\alpha^2 + \beta^2 - 2\alpha\beta) = 0$

$\Rightarrow \alpha\beta(\alpha - \beta)^2 = 0$

$\Rightarrow \alpha\beta[(\alpha + \beta)^2 - 4\alpha\beta] = 0$

$\Rightarrow \dfrac{c}{a}\left[\dfrac{b^2}{a^2} - 4 \times \dfrac{c}{a}\right] = 0$

$\Rightarrow c[b^2 - 4ac] = 0$

$\Rightarrow c\Delta = 0$

Ans. Ⓒ