TECHNO INDIA GROUP PUBLIC SCHOOL

MOCK TEST | **CLASS-XII**

MATHEMATICS

SET–2

Code: 041

Time : 3 hours | F.M.: 80

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains **38** questions. All questions are compulsory.
2. This Question paper is divided into **five** sections–**A, B, C, D** and **E**.
3. In **Section A**, Questions no. **1** to **18** are **multiple choice questions (MCQs)** with only one correct option and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In **Section B**, Questions no. **21** to **25** are **Very Short Answer (VSA)–type** questions, carrying **2 marks each**.
5. In **Section C,** Questions no. **26** to **31** are **Short Answer (SA)–type** questions, carrying **3 marks each**.
6. In **Section D,** Questions no. **32** to **35** are **Long Answer (LA)–type** questions, carrying **5 marks each**.
7. In **Section E,** Questions no. **36** to **38** are **Case study-based questions**, carrying **4 marks each**.
8. There is no overall choice. However, an internal choice has been provided in **2** questions in **Section B**, **3** questions in **Section C**, **2** questions in **Section D** and one subpart each in **2** questions of **Section E**.
9. Use of calculators is not allowed.

SECTION A

This section comprises of multiple-choice questions (MCQs) of 1 mark each.

Select the correct option (Question 1 – Question 18)

1.	The value of $\sec^2(\tan^{-1}2) + \text{cosec}^2(\cot^{-1}3)$ is equal to Ⓐ 5 Ⓑ 15 Ⓒ 13 Ⓓ None of these	[1]
2.	If $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ be such that $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ then Ⓐ $y = 2x$ Ⓑ $y = -2x$ Ⓒ $y = x$ Ⓓ $y = -x$	[1]
3.	If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$, then $pI + qA + rA^2$ Ⓐ A Ⓑ 2A Ⓒ 3A Ⓓ A^3	[1]

4. If $A = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, then Adj A = **[1]**

Ⓐ A Ⓑ A^T Ⓒ 3A Ⓓ $3A^T$

5. The inverse of the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ is **[1]**

Ⓐ $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 3 & 3 & 4 \end{bmatrix}$ Ⓑ $\begin{bmatrix} 1 & 3 & 1 \\ 4 & 3 & 8 \\ 3 & 4 & 1 \end{bmatrix}$ Ⓒ $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 3 & 4 & 3 \end{bmatrix}$ Ⓓ $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$

6. If $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$, then $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$ is equal to **[1]**

Ⓐ 0 Ⓑ abc Ⓒ –abc Ⓓ none of these

7. Let $f(x) = \begin{cases} |x|\cos\frac{1}{x} + 15x^3, & x \neq 0 \\ k, & x = 0 \end{cases}$, then f(x) is continuous at x = 0 if k is equal to **[1]**

Ⓐ 15 Ⓑ –15 Ⓒ 0 Ⓓ 6

8. If $y = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$ **[1]**

Then, find $\frac{dy}{dx}$.

Ⓐ 3 Ⓑ $\frac{3}{1+x}$ Ⓒ $-\frac{3}{1+x^2}$ Ⓓ $\frac{3}{1+x^2}$

9. Find the interval in which function f(x) = sin x + cos x, $0 \leq x \leq 2\pi$ is decreasing. **[1]**

Ⓐ $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ Ⓑ $\left(-\frac{\pi}{4}, \frac{5\pi}{4}\right)$ Ⓒ $\left(\frac{\pi}{4}, -\frac{5\pi}{4}\right)$ Ⓓ $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

10. Find the degree of the D.E $\frac{d^2y}{dx^2} + 5\cot\left(\frac{dy}{dx}\right) = 0$ **[1]**

Ⓐ five Ⓑ three Ⓒ two Ⓓ not defined

11. If $\int_0^{\pi/2} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx = \lambda$, then the value of $(2024)^{2023\lambda}$ is **[1]**

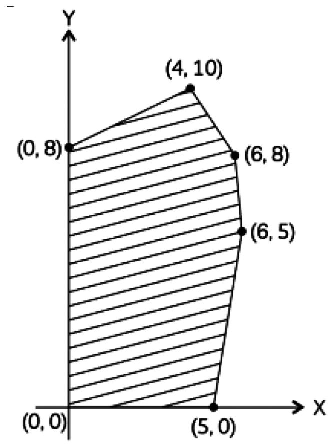
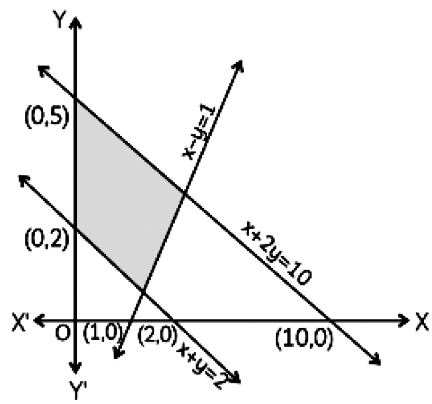
Ⓐ 0 Ⓑ 1 Ⓒ 2 Ⓓ none of these

12. If $\int \frac{\cos 4x + 1}{\cot x - \tan x} dx = k \ \cos \ 4x + c$, then **[1]**

Ⓐ $k = -\frac{1}{2}$ Ⓑ $k = -\frac{1}{8}$ Ⓒ $k = -\frac{1}{4}$ Ⓓ none of these

13. Two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides AB and AC, respectively of a ΔABC. The length of the median through A is **[1]**

Ⓐ $\frac{\sqrt{34}}{2}$ Ⓑ $\frac{\sqrt{48}}{2}$ Ⓒ $\sqrt{18}$ Ⓓ $\sqrt{34}$

14.	If the scalar projection of $\hat{i} - \lambda\hat{j} + \widehat{k}$ on $\hat{i} + 2\hat{j} + 2\widehat{k}$ is 5, then λ = Ⓐ –6 Ⓑ 6 Ⓒ 2 Ⓓ $\sqrt{6}$	[1]
15.	If $\left\|\vec{a}\right\| = 3, \left\|\vec{b}\right\| = 4$ and $\left\|\vec{a} + \vec{b}\right\| = 5$, then $\left\|\vec{a} - \vec{b}\right\|$ is equal to Ⓐ 6 Ⓑ 5 Ⓒ 4 Ⓓ 3	[1]
16.	In the given graph, the feasible region for a LPP is shaded.  The objective function Z = 2x – 3y will be minimum at: Ⓐ (4, 10) Ⓑ (6, 8) Ⓒ (0, 8) Ⓓ (6, 5)	[1]
17.	The feasible region corresponding to the linear constraints of a linear programming problem is shown in figure.  Which of the following is not a constraint to the given linear programming problem? Ⓐ $x + y \geq 2$ Ⓑ $x + 2y \leq 10$ Ⓒ $x - y \geq 1$ Ⓓ $x - y \leq 1$	[1]
18.	A bag contains 3 white, 4 black and 2 red balls. If 2 balls are drawn at random (without replacement), then the probability that both the balls are white is Ⓐ $\frac{1}{18}$ Ⓑ $\frac{1}{36}$ Ⓒ $\frac{1}{12}$ Ⓓ $\frac{1}{24}$	[1]

	Assertion Reason based Questions (19–20): **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices. (A) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (B) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A). (C) Assertion (A) is true but reason (R) is false. (D) Assertion (A) is false but reason (R) is true.	
19.	**Assertion (A):** Domain of $y = \cos^{-1}(x)$ is $[-1, 1]$ **Reason (R):** The range of the principal value branch of $y = \cos^{-1}(x)$ is $\left[0, \pi\right] - \left\{\frac{\pi}{2}\right\}$	[1]
20.	**Assertion (A):** The adjacent sides of a parallelogram are along $\vec{a} = \hat{i} + 2\hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j}$. The angle between the diagonal is 150°. **Reason (R):** Two vectors are perpendicular to each other if their dot product is zero.	[1]

SECTION B

This section comprises of 5 very short answer (VSA) type questions of 2 marks each.

21A.	Find the value of $\tan^{-1}\left(1\right) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$	[2]
	OR	
21B.	Prove $3\sin^{-1}x = \sin^{-1}\left(3x - 4x^3\right), x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$	[2]
22.	If $\sqrt{1-x^2} + \sqrt{1-y^2} = a\left(x - y\right)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.	[2]
23A.	Evaluate $I = \int e^x . \sin x \, dx$	[2]
	OR	
23B.	Find the area bounded by the curves $y = x$, $y = x^3$.	[2]
24.	Find the values of k so that the function $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3, & \text{if } x = \frac{\pi}{2} \end{cases}$ is continuous at point $x = \frac{\pi}{2}$	[2]
25.	Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\lvert\vec{a}\rvert = 3, \lvert\vec{b}\rvert = 4, \lvert\vec{c}\rvert = 5$ and each one of them being perpendicular to the sum of the other two, find $\lvert\vec{a} + \vec{b} + \vec{c}\rvert$.	[2]

SECTION C

This section comprises of 6 short answer (SA) type questions of 3 marks each.

26A.	Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$.	[3]
	OR	
26B.	Find the second order derivative of the following function: $f(x) = e^x \sin 5x$.	[3]

27.	An inverted cone has a depth of 10 cm and a base of radius 5 cm. Water is poured into it at the rate of $\frac{3}{2}$ c.c. per minute. Find the rate at which the level of water in the cone is rising when the depth is 4 cm.	[3]
28A.	Using integration, find the smaller area enclosed by the circle $x^2 + y^2 = 4$ and the line $x + y = 2$.	[3]
	OR	
28B.	Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.	[3]
29A.	Find the shortest distance between the lines $\vec{r} = \left(\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ and $\vec{r} = \left(2\hat{i} - \hat{j} - \hat{k}\right) + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$	[3]
	OR	
29B.	Find the equation of the line which intersects the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ passes through the point (1, 1, 1).	[3]
30.	Minimise $Z = 13x - 15y$ subject to the constraints $x + y \le 7$, $2x - 3y + 6 \ge 0$, $x \ge 0$ and $y \ge 0$.	[3]
31.	Probabilities of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently then, find the probability that (i) the problem is solved (ii) exactly one of them solves the problems.	[3]

SECTION D

This section comprises of 4 long answer (LA) type questions of 5 marks each.

32.	If $A = \begin{bmatrix} 3 & 1 & 2 \\ 3 & 2 & -3 \\ 2 & 0 & -1 \end{bmatrix}$, find A^{-1}. Use A^{-1} to solve the following system of equations: $3x + 3y + 2z = 1$; $x + 2y = 4$; $2x - 3y - z = 5$	[5]
33A.	Find $\int \frac{2x}{(x^2+1)(x^2+2)} dx$	[5]
	OR	
33B.	Evaluate: $\int_0^{\pi/4} \frac{dx}{1 + \tan x}$	[5]
34A.	Find the particular solution of the differential equation: $x\frac{dy}{dx} = y - x \tan\left(\frac{y}{x}\right)$, $x \neq 0$, Given that $y = \frac{\pi}{4}$, when $x = 1$.	[5]
	OR	
34B.	Find the particular solution of the differential equation: $\frac{dy}{dx} + 2y \tan x = \sin x$, given that $y = 0$ when $x = \frac{\pi}{3}$.	[5]
35.	The equations of motion of a rocket are: $x = 2t$, $y = -4t$, $z = 4t$, where the time t is given in seconds, and the coordinates of a moving point in km. What is the path of the rocket? At what distances will the rocket be from the starting point O(0, 0, 0) and from the following line in 10 seconds?	[5]

SECTION E

This section comprises of 3 case-study/passage-based questions of 4 marks each with subparts. The first two case study questions have three subparts (i), (ii), (iii) of marks 1, 1, 2 respectively. The third case study question has two subparts of 2 marks each.

36. Case Study–1

A General Election of Lok Sabha is a gigantic exercise. About 911 million people were eligible to vote and voter turnout was about 67%, the highest ever in the General Election-2019.

Let I be the set of all citizens of India who were eligible to exercise their voting right in General Elections held in 2019. A relation 'R' is defined on I as follows:

$R = \{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in General Election-2019$\}$.

Use the given data to answer the following questions:

(i) Let two friends X and Y $\in$ I. X and Y both exercised their voting right in the General Election-2019. Is $(X, Y) \in R$ true? Justify. [1]

(ii) Three brothers B_1, B_2 and B_3 exercised their voting right in General Election-2019. Then is the statement "$(B_1, B_2) \in R$, $(B_2, B_3) \in R$ and $(B_1, B_3) \in R$" true? Justify. [1]

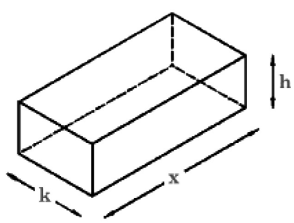
(iii) (A) Mr. 'X' and his wife 'W' both exercised their voting right in General Election-2019. Is the statement "both (X, W) and (W, X) $\in$ R" true? Justify. [2]

OR

(iii) (B) The relation $R = \{(V_1, V_2) : V_1, V_2 \in I$ and both use their voting right in General Election 2019$\}$ is symmetric and transitive but not reflexive. Justify. [2]

37. Case Study–2

A foreign client approaches ISHA BRICKS COMPANY for a special type of bricks.



The client requests for few samples of bricks as per their requirement.
The solid rectangular brick is to be made from 1 cubic feet of clay of special type.
Using above information, answer the following.

(i) According to the figure shown, the length of brick is 'x', width is 'k' and height is 'h'. Obtain an equation in terms of 'h' and 'k'. [1]

(ii) Express the surface area (A) of the brick, as a function of 'k'. [1]

(iii) (A) At what value of k, $\frac{dA}{dk} = 0$? Show that $\frac{d^2A}{dk^2}$ is positive, at this obtained value of k. What does it signify? [2]

OR

(iii) (B) Find the minimum value of A, using second derivative test. [2]

38. **Case Study–3**

An insurance company believes that people can be divided into two classes; those who are accident prone and those who are not. The company's statistics show that an accident prone person will have an accident at sometime within a fixed one-year period with probability 0.6, whereas this probability is 0.2 for a person who is not accident prone. The company knows that 20 percent of the population is accident prone.

Based on the given information, answer the following questions.

(i) What is the probability that a new policyholder will have an accident within a year of purchasing a policy? [2]

(ii) Suppose that a new policyholder has an accident within a year of purchasing a policy. What is the probability that he or she is accident prone? [2]