TECHNO INDIA GROUP PUBLIC SCHOOL

JEE MOCK TEST (Series II)

Paper Part – 3

Time : 3 hours | F.M.: 300

SOLUTION

PHYSICS

SECTION A

Section A consists of 20 questions of 4 mark each.

1. ③

$S = \boldsymbol{P} + b\boldsymbol{R} = \boldsymbol{P} + b(Q - \boldsymbol{P}) = \boldsymbol{P}(1 - b) + b\boldsymbol{Q}$

2. ②

Given, initial position of particle $r_0 = \left(2\hat{i} + 4\hat{j}\right)$ m

Initial velocity of particle, $u = \left(5\hat{i} + 4\hat{j}\right)$ m/s

Acceleration of particle, $a = \left(4\hat{i} + 5\hat{j}\right)$ m/s^2

According to second equation of motion, position of particle at time t is $r = r_0 + ut + \frac{1}{2}at^2$

At $t = 2$ s, position of particle is, $r = \left(2\hat{i} + 4\hat{j}\right) + \left(5\hat{i} + 4\hat{j}\right) \times 2 + \frac{1}{2}\left(4\hat{i} + 4\hat{j}\right) \times 4$

or, $r = (2 + 10 + 8)\hat{i} + (4 + 8 + 8)\hat{j} \Rightarrow r = 20\hat{i} + 20\hat{j}$

$\therefore$ Distance of particle from origin is, $|r| = 20\sqrt{2}$ m

3. ①

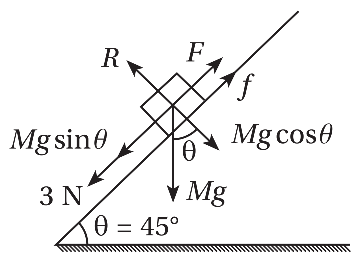
Free body diagram, for the given figure is as follows,

For the block to be in equilibrium i.e., so that it does not move downward, then $\sum f_x = 0$

$\therefore 3 + Mg\sin\theta - F - f = 0$ or, $3 + Mg\sin\theta = F + f$

As, frictional force, $f = \mu R$

$\therefore 3 + Mg\sin\theta = F + \mu R$ ... (i)



Similarly, $\sum f_y = 0$ $-Mg\cos\theta + R = 0$

or, $Mg\cos\theta = R$ ... (ii)

Substituting the value of 'R' from Eq. (ii) to Eq. (i), we get

$3 + Mg\sin\theta = F + \mu(Mg\cos\theta)$ ... (iii)

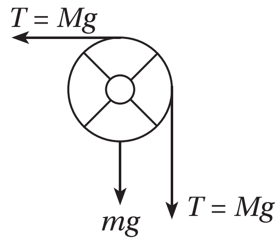
Here, $M = 10$ kg, $\theta = 45°$, $g = 10$ m/s^2 and $\mu = 0.6$

Substituting these values is Eq. (iii), we get $3 + (10 \times 10\sin 45°) - (0.6 \times 10 \times 10\cos 45°) = F$

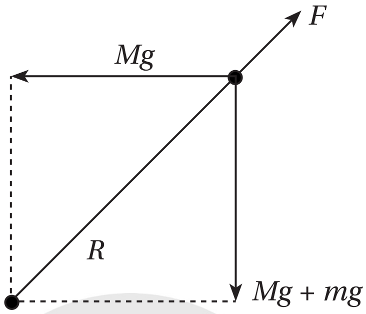
$\Rightarrow F = 3 + \frac{100}{\sqrt{2}} - \frac{60}{\sqrt{2}} = 3 + \frac{40}{\sqrt{2}} = 3 + 20\sqrt{2} = 31.8$ N or, $F \simeq 32$ N

4. ④

Free body diagram of pulley is shown in figure. Pulley is in equilibrium under four forces. Three forces as shown in figure and the fourth, which is equal and opposite to the resultant of these three forces, is the force applied by the clamp on the pulley (say F). Resultant R of these three forces is $R=\left(\sqrt{(M+m)^2+M^2}\right)g$



Therefore, the force F is equal and opposite to R as shown in figure.



$\therefore F=\left(\sqrt{(M+m)^2+M^2}\right)g$

5. ②

Height fallen up to point Q

$$h=R\sin 30°=40\times\frac{1}{2}=20 \text{ m}$$

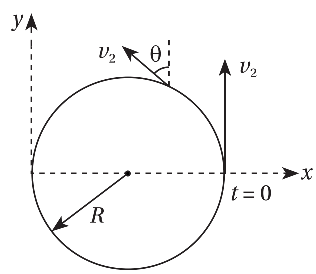
Work done against friction = Initial mechanical energy – Final mechanical energy $=mgh-\frac{1}{2}mv^2$

Putting the values, we get

$$150=1\times10\times20-\frac{1}{2}\times1\times v^2 \quad \Rightarrow v=10 \text{ m/s}$$

6. ①

Angular speed of particle about centre of the circle



$$\omega=\frac{v_2}{R}, \theta=\omega t=\frac{v_2}{R}t$$

$$\boldsymbol{v}_p=\left(-v_2\sin\theta\hat{i}+v_2\cos\theta\hat{j}\right)$$

or, $\boldsymbol{v}_p=\left(-v_2\sin\frac{v_2}{R}t\hat{i}+v_2\cos\frac{v_2}{R}t\hat{j}\right)$

and $v_m=v_1\hat{j}$

$\therefore$ Linear momentum of particle w.r.t. man as a function of time is

$$L_{pm}=m\left(\boldsymbol{v}_p-v_m\right)=m\left[\left(-v_2\sin\frac{v_2}{R}t\right)\hat{i}+\left(v_2\cos\frac{v_2}{R}t-v_1\right)\hat{j}\right]$$

7. ②

$$\Rightarrow \frac{1}{2}mv_1^2 = 0.36\times\frac{1}{2}mv^2$$

$\Rightarrow v_1 = 0.6\,v$ … (i)

As unknown nucleus gained 64% of energy of α, we have

$$\frac{1}{2}Mv_2^2 = 0.64\times\frac{1}{2}mv^2$$

$$\therefore\ v_2 = \sqrt{\frac{m}{M}}\times 0.8\,v \qquad \text{… (ii)}$$

From momentum conservation, we have

$$mv = Mv_2 - mv_1$$

Substituting values of v_1 and v_2 from Eqs. (i) and (ii), we have

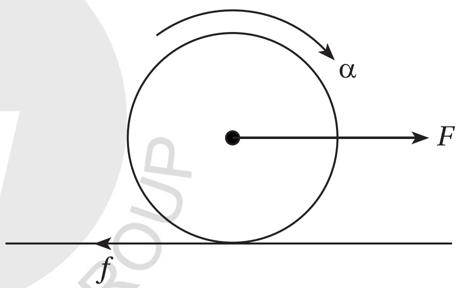
$$mv = M\sqrt{\frac{m}{M}}\times 0.8v - m\times 0.6v$$

$$\Rightarrow 1.6mv = \sqrt{mM}\times 0.8v \qquad \Rightarrow 2m = \sqrt{mM} \qquad \Rightarrow 4m^2 = mM \quad \Rightarrow M = 4m$$

8. ②

When force F is applied at the centre of roller of mass m as shown in the figure below

Its acceleration is given by



$$\frac{(F-f)}{m} = a \qquad \text{… (i)}$$

where, f = force of friction and m = mass of roller.

Torque on roller is provided by friction f and it is

$$\tau = fR = I\alpha \qquad \text{… (ii)}$$

where, I = moment of inertia of solid cylindrical roller.

$$= \frac{mR^2}{2}$$

and α = angular acceleration of cylinder = a/R

$$\text{Hence,}\quad \tau = \frac{mR^2}{2}\cdot\frac{a}{R} = \frac{maR}{2}$$

From Eq. (ii), ($\tau = fR$)

$$f = \frac{ma}{2} \qquad \text{… (iii)}$$

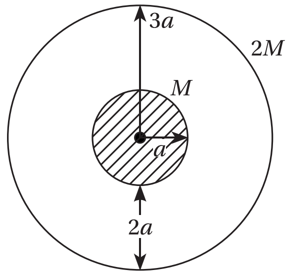
From Eqs. (i) and (iii), we get

$$F = \frac{3}{2}ma \quad \Rightarrow a = \frac{2F}{3m}$$

$$\text{So,}\quad \alpha = \frac{2F}{3mR}$$

9. ③

According to question, diagram will be as follows



Gravitational field due to solid sphere of radius a at a distance, $r = 3a$ i.e., $(r > R)$ is

$$E_1 = \frac{GM}{r^2} = \frac{GM}{(3a)^2} = \frac{GM}{9a^2}$$

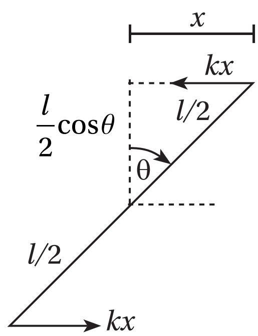
Similarly, gravitational field due to spherical shell at a distance, $r = 3a$,

i.e., $$E_2 = \frac{GM}{R^2} = \frac{G(2M)}{(3a)^2} = \frac{2GM}{9a^2}$$

Both fields are attractive in nature, so direction will be same.

Net field, $$E_{net} = \frac{GM}{9a^2} + \frac{2GM}{9a^2} = \frac{GM}{3a^2}$$

10. ③



$$\tau = (kx)l\cos\theta = \frac{ml^2}{12} \times \alpha \qquad \theta \to \text{small};\ \cos\theta = 1;\quad x = \frac{l}{2}\theta$$

$$\therefore \frac{kl^2}{2}\cdot\theta = \frac{ml^2}{12}\cdot\alpha \qquad \alpha = \left(\frac{6k}{m}\right)\theta = \omega^2\theta$$

$$\omega = \sqrt{6k/m} = 2\pi f \qquad f = \frac{1}{2\pi}\cdot\sqrt{\frac{6k}{m}}$$

11. ④

Mass per unit time of a liquid flow is given by

$$\frac{dm}{dt} = \rho A v$$

where, ρ is density of liquid, A is area through which it is flowing and v is velocity.

∴ Rate of change in momentum of the 25% of liquid which loses all momentum is

$$\frac{dp_1}{dt} = \frac{1}{4}\left(\frac{dm}{dt}\right)v = \frac{1}{4}\rho A v^2 \qquad \dots \text{(i)}$$

and the rate of change in momentum of the 25% of the liquid which comes back with same speed.

$$\frac{dp_2}{dt} = \frac{1}{4}\left(\frac{dm}{dt}\right)\times 2v = \frac{1}{2}\rho A v^2 \qquad \dots \text{(ii)}$$

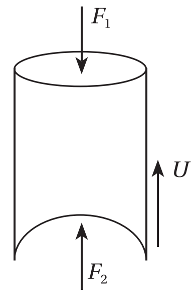
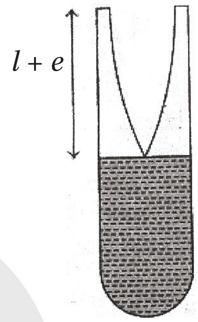
[∵ Net change in velocity is = $2v$]

∴ Net pressure on the mesh is

$$p = \frac{F_{net}}{A} = \frac{(dp_1/dt + dp_2/dt)}{A} \qquad \left[\because F = \frac{dp}{dt}\right]$$

From Eqs. (i) and (ii), we get

$$p = \frac{3}{4}\rho v^2 A / A = \frac{3}{4}\rho v^2$$

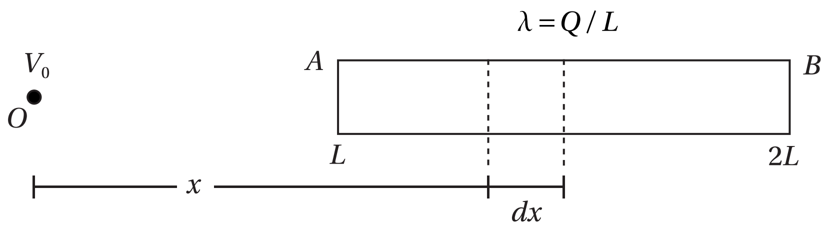
12.	④ $F_2 - F_1 = U$ $F_2 = F_1 + U = (P_0 + h\rho g)\pi R^2 + V\rho g$ $= P_0\pi R^2 + \rho g(V + \pi R^2 h) =$ by atm+by liquid  	
13.	① In first resonance, length of air column $= \frac{\lambda}{4}$ So, $l_1 + e = \frac{\lambda}{4}$ or, $11 \times 4 + 4e = \lambda$ So, speed of sound is $\Rightarrow v = f_1\lambda = 512(44 + 4e)$ And in second case, $l_1' + e = \frac{\lambda'}{4}$ or, $27 \times 4 + 4e = \lambda'$ $\Rightarrow v = f_2\lambda' = 256(108 + 4e)$ Dividing both Eqs. (i) and (ii), we get $1 = \frac{512(44+4e)}{256(108+4e)} \Rightarrow e = 5$ cm Substituting value of e in Eq. (i), we get Speed of sound $v = 512(44 + 4e) = 512(44 + 4 \times 5) = 512 \times 64$ cm s^{-1} = 327.68 ms^{-1} ≈ 328 ms^{-1}  	
14.	④ Let interface temperature in steady state conduction is θ, then assuming no heat loss through sides; Rate of heat flow through first slab = Rate of heat flow through second slab $\Rightarrow \frac{(3K)A(\theta_2 - \theta)}{d} = \frac{KA(\theta - \theta_1)}{3d} \Rightarrow 9(\theta_2 - \theta) = \theta - \theta_1 \Rightarrow 9\theta_2 + \theta_1 = 10\theta \Rightarrow \theta = \frac{9}{10}\theta_2 + \frac{1}{10}\theta_1$	
15.	② ① p–V graph is not rectangular hyperbola. Therefore, process A–B is not isothermal. ② In process BCD, product of pV (therefore temperature and internal energy) is decreasing Further, volume is decreasing. Hence, work done is also negative. Hence, Q will be negative or heat will flow out of the gas. ③ W_{ABC} = positive ④ For clockwise cycle on p–V diagram with P on Y-axis, net work done is positive.	

16.	④ $\frac{1}{f_1}=(\mu_1-1)\left[\frac{1}{\infty}-\frac{1}{-R}\right]=\frac{\mu_1-1}{R}$ $\frac{1}{f_2}=(\mu_2-1)\left[-\frac{1}{R}-\frac{1}{\infty}\right]=-\frac{(\mu_2-1)}{R}$ $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{\mu_1-1}{R}-\frac{\mu_2-1}{R}=\frac{\mu_1-\mu_2}{R}$ $\therefore F=\frac{R}{\mu_1-\mu_2}$	
17.	③ $\sqrt{4d^2+d^2}-2d=\sqrt{5}d-2d=\left(\sqrt{5}-2\right)d=\frac{\lambda}{2}$ $\therefore d=\frac{\lambda}{2\left(\sqrt{5}-2\right)}$	
18.	④ $i_1+i_2=i$ $\frac{20-V_c}{2}+\frac{10-V_c}{4}=\frac{V_c-0}{2}\ \Rightarrow \frac{40-2V_c+10-V_c}{4}=\frac{V_c}{2}\ \Rightarrow 50-3V_c=2V_c$ V_c = 10 Volt $i=V_c/2$ = 5 A	
19.	④ $Q=CE$; $V=$ Common potential $=\frac{Q}{C+3C}=\frac{Q}{4C}=\frac{E}{4}$ Energy stored $=\frac{1}{2}\cdot C\cdot V^2+\frac{1}{2}3CV^2=2CV^2=2\cdot C\cdot\frac{Q^2}{16C^2}=\frac{Q^2}{8C}$ Energy dissipated $=\frac{Q^2}{2C}-\frac{Q^2}{8C}=\frac{3Q^2}{8C}$	
20.	④ $\vec{F}=I\left(\overrightarrow{PQ}\times B\right)=I\left[\left\{2(L+R)\hat{i}\right\}\times\vec{B}\right]$	

SECTION B

Section B consists of 5 questions of 4 marks each.

21. 9;



$$dV = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Q/L)dx}{x}$$

$$V_0 = \frac{1}{4\pi\varepsilon_0}\int_L^{2L}\frac{Q}{L}\cdot\frac{dx}{x} = \frac{Q}{4\pi\varepsilon_0 L}\cdot\ln 2$$

$$V_0\left(\frac{L}{Q\ln 2}\right)\times 10^{-9} = 9$$

22. 12;

$V_a - (5)(3) - (1)(3) + 2\times 3 = V_b$

$V_a - V_b = 12$

23. 3;

$$W_E + W_B = \frac{1}{2}m\left(4V^2 - V^2\right)$$

$$(qE)(2a\cos 0^\circ) = \frac{1}{2}m\cdot 3v^2$$

$$\left[\frac{4qEa}{mv^2}\right] = 3$$

24. 15;

$V_A - (5)(1) + 15 + (5\times 10^{-3})(10^{3}) = V_B$

$V_B - V_A = 15$ Volt

25. 4;

Case I : V0 = 12 – 0.3 = 11.7 volt

Case II : V0 = 12 – 0.7 = 11.3 volt

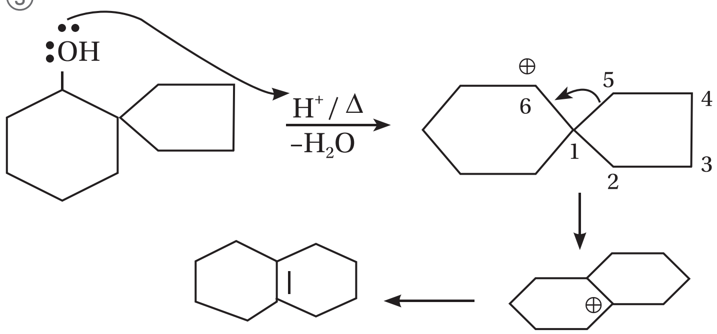
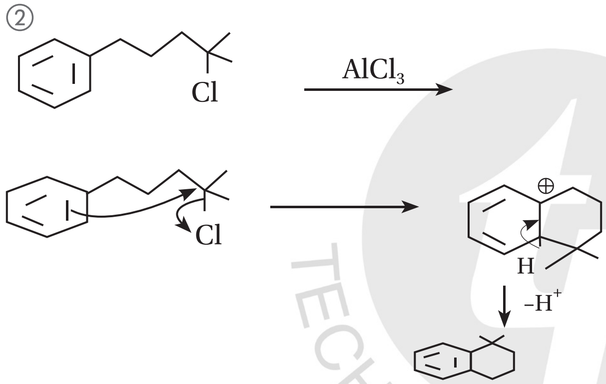
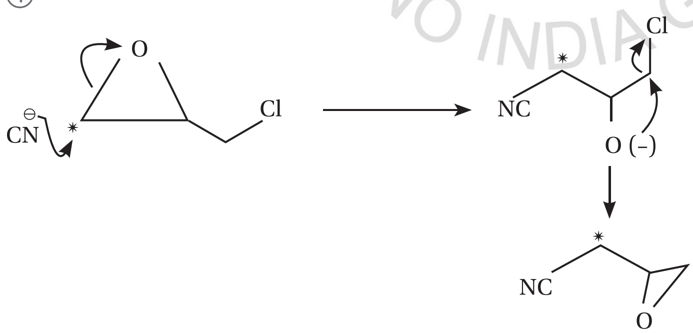
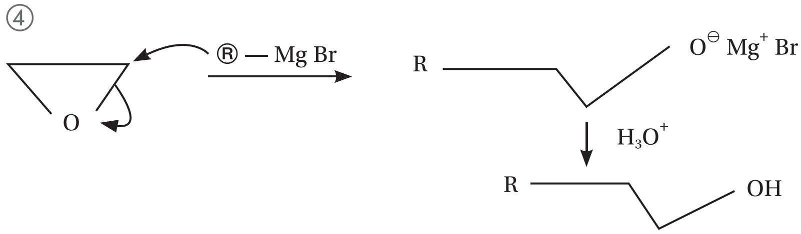
$\Delta V_0 = 11.7 - 11.3 = 0.4$ volt

$10\times\Delta V_0 = 4$ volt

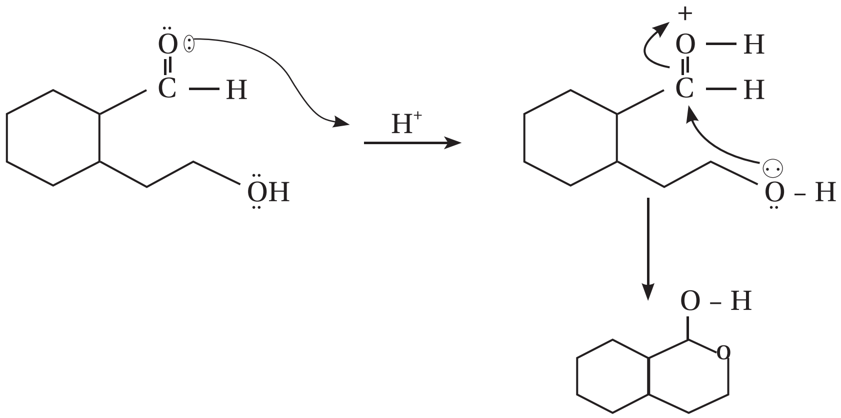
CHEMISTRY

SECTION A

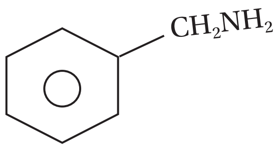
Section A consists of 20 questions of 4 mark each.

26.	(3) 	
27.	(2) 	
28.	(4) 	
29.	(4) 	
30.	(3) 3–methyl pent –1 – en – 4 – yne	

31. ④



32. ③



(Benzylamine)

33. ③

Due to 'd' – orbital resonance 'A' is more acidic than 'B' ; 'F' has no vacant 'd' orbital while 'Cl' has vacant d-oirbital.

34. ②

MP. A > B > C

MP : A = 256 k

B = 249 k

C = 323 k

35. ①

$CH_3Br + AgF \longrightarrow CH_3F + AgBr$ Swarts reaction ?

36. ③

BE : A > B > C > D ; D M : B > A > C > D

Example :	CH_3F	CH_3Cl	CH_3Br	CH_3I
B. E. (KJ/ mole)	452	351	293	234
D. M. (Debye)	1.847	1.860	1.83	1.636

37. ②

5.8×10^{-10} m.

$\Delta v = 0.02 \times \text{velocity} = 0.02 \times 5 \times 10^6 = 10^5$ m.

$$\Delta x \times \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x . m \Delta p = \frac{h}{4\pi}$$

$$\Delta x = \frac{h}{4\pi m \times \Delta v} = \frac{6.6 \times 10^{-34} \text{ J-s.}}{4 \times 3.14 \times 10^5 \times 9.1 \times 10^{-31} \text{kg}}$$

$= 5.8 \times 10^{-10}$ m

38. ②

$NaOH + H_3PO_4 \rightarrow N_2HPO_4 + H_2O$
only one hydrogen is only displaced
So, n factor of $H_3PO_4 = 1$

$\therefore$ Eqv.wt of H_3PO_4 = M.W of $\frac{H_3PO_4}{1} = \frac{98}{1} = 98$

M. W of H_3PO_4
$= 3 + 31 + 64$
$= 98$

39. ②

There is negative deviation from Raoult's Law.

$PT = P^O_A X_A + P^O_B X_B$

$= 150 \times \frac{1}{3} + 240 \times \frac{2}{3}$

$= 50 + 160 = 210$ torr.

$\therefore P_{EXP} < P_{calculated}$

Thus, there is a negative deviation from Raoult's Law.

40. ④

$[Cr(SCN)_2 (NH_3)_4]^{2+}$
Ma_2b_4 complex exhibit cis and trans isomerism. $SCN^{\ominus}$ is ambidentate ligand. So, it exhibit linkage isomerism.

41. ③

$[V(H_2O)_6]^{3+}$
V = 23 (At. No.)
E. C : of V_{23} = (Av] $4s^2 3d^3$
V^{3+} ion = [Ar] $3d^2$
= [↿][↿][ ][ ][ ]
n = 2 = no. of unpaired electron.

$\therefore$ Magnettic Moment, $\mu = \sqrt{n(n+2)}$ B.M
$= \sqrt{2(2+2)}$ B.M
= 2.83 B. M.

42. ④

Hg S ; HgS soluble only in aquaregia.

43. ①

SO_3^{2-} ion only

$$Na_2SO_3 + H_2SO_4 \longrightarrow Na_2SO_4 + H_2O + SO_2\uparrow$$

$$Ca(OH)_2 + SO_2 \longrightarrow \underset{\text{(White turbidity)}}{CaSO_3\downarrow} + H_2O$$

$$CaSO_3\downarrow + SO_2 + H_2O \longrightarrow \underset{\text{(Soluble)}}{Ca\,(HSO_3)_2}$$

44. ③

$$Cu^{++} + 2e^{\ominus} \longrightarrow Cu$$

$$2\,H_2O \longrightarrow O_2 + 4H^+ + 4e^{\ominus}$$

Eq. of H^+ formed = Eq. of Cu^{2+} lost

$$= \frac{i \times t}{96500} = \frac{96.5 \times 10}{96500} = 0.01$$

$\therefore$ $[H^+] = \frac{0.01}{1} = 10^{-2}$

$\therefore$ $pH = -\log_{10}[H+] = -\log_{10}10^{-2} = +2$.

45. ②

$\frac{k_1}{(k_1 + k_2)} \times 50$;

$$A\,(s) \begin{cases} \xrightarrow{K_1} B\,(g) + C\,(g) \\ \xrightarrow{K_2} B\,(g) + E\,(g) \end{cases}$$

Fraction of B = Fraction of $C = \frac{k_1}{k_1 + k_2}$

Fraction of D = Fraction of $E = \frac{k_2}{k_1 + k_2}$

Total gaseous phase = (B + C) + (D + E)

$$= \frac{2\,k_1}{(k_1 + k_1)} + \frac{2\,k_2}{(k_1 + k_2)} = 2.$$

$\therefore$ Mol % of B = $\frac{k_1}{(k_1 + k_2)} \times \frac{100}{2} = \frac{50k_1}{(k_1 + k_2)}$

SECTION B

Section B consists of 5 questions of 4 marks each.

46. 5;

$E_{un} = E_{\,OPQH} + E_{RP\longrightarrow}$ calomel

$$= \left(E^{\circ}_{OPQH} - \frac{0.059}{2}\log\left[H^{+}\right]^{2}\right) + E^{\circ}_{RP_{Cal}}$$

$-0.124 = -0.699 + 0.059 \log_{10}(H^{+}) + 0.280$

$\Rightarrow +0.124 = 0.419 - 0.059\ pH$

$\Rightarrow pH = 5$.

47. 6;

he isoelectric point of amino acid $= \dfrac{pk_{a_1} + pk_{a_2}}{2}$

$= \dfrac{2.3 + 9.7}{2}$

$= \dfrac{12.0}{2}$

$= 6$

48. 3;

$[H+] = c\alpha = 0.1 \times 0.01 = 10^{-3}$

$\therefore\ pH = -\log_{10}[H^{+}]$

or $pH = -\log_{10}10^{-3}$

$= +3.$

49. 6;

$q_v = 0.2 \times 6 \times 15 = 18$ cal.

$\therefore$ 0.2 mole requires 18 cal

$\therefore$ 1 mole will require $= \dfrac{18}{0.2} = 90\,\text{Cal}$

1 mole of 15°C rise requires 90 cal

1 mole of 15°C rise require $\dfrac{90}{15} = 6$

$\therefore$ Cv = 6 Cal.

50. 8 ;

Total no. of geometrical no.s $= 2^{n} = 2^{3} = 8$

n = no. of c = c = 3, when both end different atom or group.

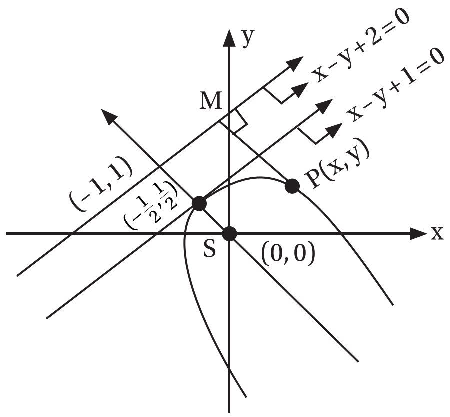
MATHEMATICS

SECTION A

Section A consists of 20 questions of 4 mark each.

51. ③

PS = PM



$$\sqrt{x^2 + y^2} = \left|\frac{x - y + 2}{\sqrt{(1)^2 + (-1)^2}}\right|$$

$\Rightarrow\ x^2 + y^2 + 2xy - 4x + 4y + 4 = 0$

52. ②

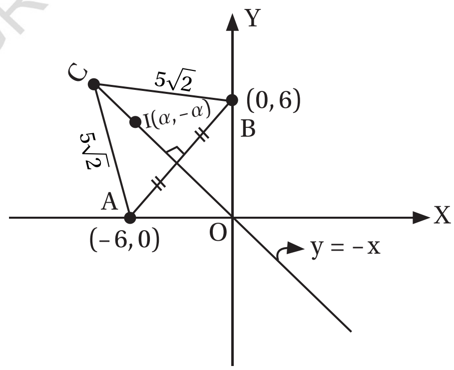
AB : $x - y + 6 = 0$

AC : $7x + y + 42 = 0$

$$\left|\frac{6\alpha + 42}{5\sqrt{2}}\right| = \left|\frac{2\alpha + 6}{\sqrt{2}}\right| \Rightarrow \alpha = -\frac{9}{2}$$

$$I\left(-\frac{9}{2}, \frac{9}{2}\right) \text{ radius } = \frac{3}{\sqrt{2}}$$

Circle: $\left(x + \frac{9}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = \left(\frac{3}{\sqrt{2}}\right)^2 \Rightarrow x^2 + y^2 + 9x - 9y + 36 = 0$



53. ②

$$e = \sqrt{1 - \frac{60}{90}} = \frac{1}{\sqrt{3}}$$

54. ②

$f(x) = 1\ \forall\ x \in R$

$g(f(x)) = g(1) < 0$

$\Rightarrow\ K^2 - 11K + 24 < 0$

$\Rightarrow\ 3 < K < 8 \Rightarrow K = 4, 5, 6, 7$

55.	③ $\frac{6!}{2!2!2!}=90$
56.	① Let, $I=\lim\limits_{x\to 0} x^x \Rightarrow \log_e^I = \lim\limits_{x\to 0^+} x\log_e^x = \lim\limits_{x\to 0^+}\frac{\log_e^x}{\frac{1}{x}} = \frac{\frac{1}{x}}{\frac{1}{x^2}} = 0$ $\therefore\ I = e^\circ = 1$ Let $P=\lim\limits_{x\to 0^+} x^{(x^x-1)} \Rightarrow \log_e^p = \lim\limits_{x\to 0^+}(x^x-1)\log_e^x = \lim\limits_{x\to 0}\frac{e^{x\log_e^x}-1}{x\log x}.x(\log_e^x)^2$ $\therefore\ P = 1$ $q = \lim\limits_{x\to 0^+}(x^{x^x}-1) = \lim\limits_{x\to 0^+}(x^1-1) = 0$ $= -1 \quad \therefore\ p - q = 2$
57.	① 3 alike, 3 distinct $\longrightarrow {}^4C_1\times{}^3C_3\times\frac{6!}{3!}=480$ 2 alike, 2dlike, 2distinct $\longrightarrow {}^4C_2\times{}^2C_2\times\frac{6!}{2!2!}=1080$ Total = 1080 + 480 = 1560.
58.	② adj (AB) = adj B . adj A adj $(A^{-1}) = (\text{adj } A)^{-1}$ adj $(Q^{-1}BP^{-1})$ = adj (P^{-1}) . adj B . adj (Q^{-1}) $= \|P\| . P A \|Q\| . Q$ $= PAQ\ (\because\ \|P\| = \|Q\| = 1)$
59.	② $\cos^{-1}(2x^2-1) = 2\pi - 2\cos^{-1}x \quad (\because\ x<0)$ $\cos^{-1}(2x^2-1) - 2\sin^{-1}x = 2\pi - 2\cos^{-1}x - 2\sin^{-1}x$ $= 2\pi - 2(\cos^{-1}x + \sin^{-1}x)$ $= 2\pi - 2.\frac{\pi}{2}$ $= \pi$
60.	① $I=\int_{\frac{1}{2}}^{2}\left(1-\frac{1}{x^2}\right)\log_e\left(\frac{1-x+x^2}{1+x+x^2}\right)dx = \int_{\frac{1}{2}}^{2}\frac{(1-x^2)}{x^2}\log\frac{(x^2-x+1)}{(x^2+x+1)}dx,\ \left(\text{put } x=\frac{1}{t}\right)$ ... (1) ... (2) (1) + (2) $\Rightarrow$ 2I = O $\Rightarrow$ I = 0

61. ③

$Z^3 + aZ^2 + bZ + c = 0 \rightarrow Z_1, Z_2, Z_3$

$Z_1 + Z_2 + Z_3 = -a$

$Z_1Z_2 + Z_2Z_3 + Z_3Z_1 = b$

$Z_1 . Z_2 . Z_3 = -c$

$|-a| = |Z_1 + Z_2 + Z_3| \le |Z_1| + |Z_2| + |Z_3| = 3 \Rightarrow |a| \le 3$

$|b| = |Z_1Z_2 + Z_2Z_3 + Z_3Z_1| \le |Z_1Z_2| + |Z_2Z_3| + |Z_3Z_1| = 3 \Rightarrow |b| \le 3$

$|-c| = |Z_1Z_2Z_3| \Rightarrow |c| = 1$

$|3-4i + a| \le |3-4i| + a = 5 + |a| \le 5 + 3 = 8$

62. ①

$A = \left\{x \in R : \frac{x-1}{x} > 1\right\}$ $\qquad l_n(x^2 - 4x + 4) \ge 0$

$\frac{x-1}{x} > 1$ $\qquad x^2 - 4x + 4 \ge 1$

$\Rightarrow \frac{1}{x} < 0$ $\qquad \Rightarrow (x-3)(x-1) \ge 0$

$\Rightarrow x < 0 \quad \dots (1)$ $\qquad \Rightarrow (x \le 1) \cup (x \ge 3) \quad \dots (2)$

$(1) \cap (2) = (-\infty, 0)$

63. ②

$I = \int x^5(1+x^3)^{2/3}dx = A(1+x^3)^{8/3} + B(1+x^3)^{5/3} + C$

Put $x^3 = t$

$I = \frac{1}{3}\int t(1+t)^{\frac{2}{3}}dt$

$= \frac{1}{8}(t+1)^{\frac{8}{3}} - \frac{1}{5}(t+1)^{\frac{5}{3}} + C$

$\therefore A = \frac{1}{8}, B = -\frac{1}{5}$

64. ③

$\sum_{j=1}^{21} a_j = 693 = \frac{21}{2}(a_1 + a_{21}) \Rightarrow a_1 + a_{21} = 66 \quad \dots (1)$

$a_{11} = AM = \frac{693}{21} = 33 \quad \dots (2)$

$a_2 + a_{20} = a_3 + a_{19} + \dots + a_{10} + a_{12}$

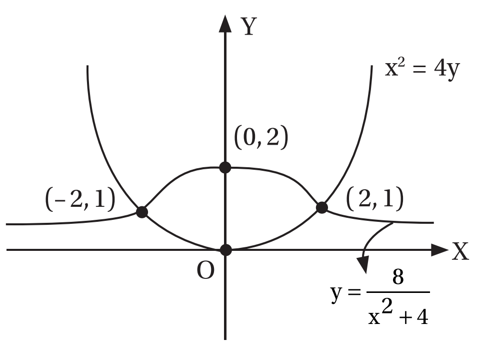
$\sum_{i=0}^{10} a_{2i} + 1 = 5(a_1 + a_{21}) + a_{11} = 363$

65.	② $A=\int_0^{\pi}\frac{\cos x}{(x+2)^2}dx$ $B=\int_0^{\pi/2}\frac{\sin 2x}{(x+1)}dx$ put $2x=t \Rightarrow B=\int_0^{\pi}\frac{\sin t}{(t+2)}dt$ $=\left[\frac{-1}{t+2}(\cos t)\right]_0^{\pi}-\int_0^{\pi}\frac{-1\times(-\sin t)dt}{(t+2)^2}$ (using IBP) $B=\frac{1}{\pi+2}+\frac{1}{2}-A$
66.	② $x + 3y + 1 = 0$ ... L_1 $Kx + 2y - 2 = 0$ ... L_2 $2x - y + 3 = 0$ ... L_3 If $L_1 \parallel L_2$ = $K=\frac{2}{3}$ If $L_2 \parallel L_3$ = $K = -4$ If L_1, L_2, L_3 are concurrent then $\begin{vmatrix} 1 & 3 & 1 \\ K & 2 & -2 \\ 2 & -1 & 3 \end{vmatrix} = 0 \Rightarrow K=-\frac{6}{5}$ $\therefore\ K=R-\left\{-4,-\frac{6}{5},\frac{2}{3}\right\}$
67.	③ $x=\sec^2\theta$ $y=\cot\theta$ $\sec^2\theta-\tan^2\theta=1$ $\Rightarrow\ x-\frac{1}{y^2}=1$ $\Rightarrow\ y^2=\frac{1}{x-1}$ ... (1) $\left.\frac{dy}{dx}\right\|_{(2,1)}=-\frac{1}{2}$ ... (2) Tangent at (2, 1) : $y=-\frac{1}{2}x+2$ ... (3) $(1)\cap(3) \Rightarrow x=2, y=1;\ x=5, y=-\frac{1}{2}$ Now $P=(2,1)$ given $\therefore\ Q=(5,-\frac{1}{2})$ $\therefore\ PQ=\frac{3\sqrt{5}}{2}$

68.	④ Let $\vec{V}=(\vec{c}\times\vec{d})\times(\vec{a}\times\vec{b})+(\vec{a}\times\vec{c})\times(\vec{d}\times\vec{b})-(\vec{b}\times\vec{c})\times(\vec{a}\times\vec{d})$ $=-[\vec{c}\,\vec{d}\,\vec{b}]\vec{a}+[\vec{c}\,\vec{d}\,\vec{a}]\vec{b}+[\vec{a}\,\vec{c}\,\vec{b}]\vec{d}-[\vec{a}\,\vec{c}\,\vec{d}]\vec{b}\;-[\vec{b}\,\vec{c}\,\vec{d}]\vec{a}+[\vec{b}\,\vec{c}\,\vec{a}]\vec{d}$ $(\because [\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]=[\vec{\beta}\,\vec{\gamma}\,\vec{\alpha}]=[\vec{\gamma}\,\vec{\alpha}\,\vec{\beta}])$ $=-2[\vec{b}\,\vec{c}\,\vec{d}]\vec{a}$ $=\lambda\vec{a}$ where $\lambda=-2[\vec{b}\,\vec{c}\,\vec{d}]$ a scalar $\therefore\ \vec{V}\parallel\vec{a}$
69.	① $\overline{P\cup(\bar{P}\cup Q)}$ $=\bar{P}\cap(P\cap\bar{Q})$ $=(P\cap\bar{Q})\cap\bar{P}$ $=(P\wedge\sim Q)\sim P$
70.	② $\left(x\frac{dy}{dx}+y\right)=\frac{e^{xy}}{x^2}\left(x\frac{dy}{dx}-y\right)$ $\Rightarrow e^{-xy}(xdy+ydx)=\frac{xdy-ydx}{x^2}$ $\Rightarrow \int e^{-xy}d(xy)=\int d\left(\frac{y}{x}\right)$ $\Rightarrow \frac{e^{-xy}}{-1}=\frac{y}{x}+c_1 \Rightarrow \frac{y}{x}+e^{-xy}=c$

SECTION B

Section B consists of 5 questions of 4 marks each.

71.	(4) $I=2\int_0^2\left(\frac{8}{x^2+4}-\frac{x^2}{4}\right)dx=2\pi-\frac{4}{3}=k$  $[K]=\left[2\pi-\frac{4}{3}\right]=4$

72.	(8) $(A + I)^3 + (A - I)^3 - 6A = B$ $\Rightarrow\ B = 2A^3$ $\Rightarrow\ B = 2I$ $\Rightarrow\ \lvert B \rvert = 2^3 = 8 \qquad (\because\ \lvert I \rvert = 1)$
73.	(788) Red Box — Green Box 5R — 1R8G $\rightarrow$ Favourable 4R1G — 2R7G 3R2G — 3R6G 2R3G — 4R5G 1R4G — 5R4G $\rightarrow$ favourable 5G — 6R3G $P = \dfrac{{}^6C_5 + {}^6C_1 \times {}^8C_4}{{}^{14}C_5} = \dfrac{213}{2001} = \dfrac{p}{q}$; q – p = 788
74.	(6) $\left[\dfrac{x^2+1}{p}\right] = 0\ \forall\ x \in [-2, 2] \Rightarrow 0 \le \dfrac{x^2+1}{p} < 1 \Rightarrow \dfrac{5}{p} < 1 \Rightarrow p > 5$ least value of P is 6.
75.	(3) $\left(\dfrac{x+1}{x^{2/3} - x^{1/3} + 1} - \dfrac{x-1}{x - x^{1/2}} + \dfrac{x-1}{x^{3/2} - x^{1/2}}\right)^9$ $= \left(x^{1/3} + 1 - (x^{-1/2} + 1) + x^{-1/2}\right)^9$ $= \left(x^{1/3} + \cancel{1} - \cancel{x^{-1/2}} - \cancel{1} + \cancel{x^{-1/2}}\right)^9$ $= \left(x^{1/3}\right)^9$ $= x^3$ So, highest power of x is 3