# TECHNO INDIA GROUP PUBLIC SCHOOL

## JEE MOCK TEST (Series II)

### Paper Part – 1

Time : 3 hours

# SOLUTION

F.M.: 300

# PHYSICS

## SECTION A

**Section A consists of 20 questions of 4 mark each.**

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1. ②

$$x=\frac{(1-y)}{(1+y)} \quad \Rightarrow \ln x=\ln(1-y)-\ln(1+y)$$

$$\frac{dx}{x}=-\frac{\Delta y}{1-y}-\frac{\Delta y}{1+y}=-\Delta y\left[\frac{1+y+1-y}{1-y^2}\right]=-\frac{2\Delta y}{1-y^2}$$

$$\Delta x=-\frac{2\Delta y}{1-y^2}\cdot x=-\frac{2\Delta y}{(1-y)(1+y)}\cdot\frac{(1-y)}{(1+y)}=\frac{-2\Delta y}{(1+y)^2}$$

2. ②

for $m$ = 1 kg (or unit mass)

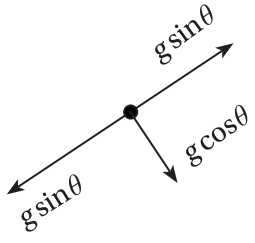
$$\int_{v_0}^{0}\frac{dv}{g+mkv}=-\int_{0}^{t}dt \quad \Rightarrow \quad \frac{1}{mk}\left[\ln(g+mkv)\right]_{v_0}^{0}=-t$$

$$\ln(g)-\ln(g+mkv_0)=-mkt \quad \Rightarrow \quad \ln\frac{g}{g+mkv_0}=-mkt$$

$$\therefore t=\frac{1}{mk}\ln\left(\frac{g+mkv_0}{g}\right) \qquad \therefore t=\frac{1}{mk}\ln\left[1+\left(\frac{mk}{g}\right)v_0\right] \quad (\text{put } m=1)$$

3. ③

$$PQ=\text{Range}=\frac{u^2\sin 2\alpha}{g\cos\theta}$$

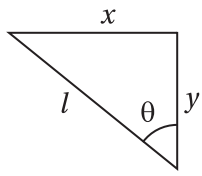


4. ④

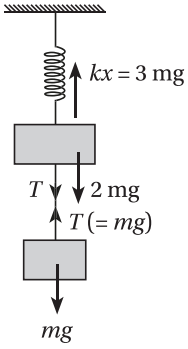
$l^2 = x^2 + y^2$

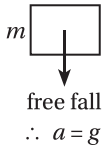
$2l\frac{dl}{dt} = 2y\frac{dy}{dt}$

$\frac{dy}{dt} = \frac{u}{(y/l)} = \frac{u}{\cos\theta}$

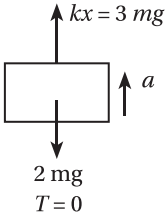



5. ①



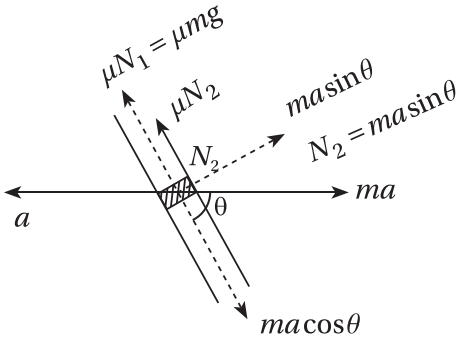



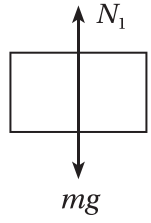

Withdrawal of force in spring is not immediate




$3mg - 2mg = (2m)a$ $\quad mg = (2m)a$ $\quad a = g/2\ (\uparrow)$

6. ②




$F_{net} = ma\cos\theta - \mu N_1 - \mu N_2 = ma\cos\theta - \mu mg - \mu ma\sin\theta$

$a_{rel} = a\cos\theta - \mu g - \mu a\sin\theta = 10\ \text{m/s}^2$

7. ④

$m\frac{dv}{dt}=6t \qquad m=1$

$(m)\int_0^v dv=6\int_0^1 t\,dt$

$m\cdot v=6\cdot\left[\frac{t^2}{2}\right]_0^1=\frac{6}{2}=3$

$k=\frac{P^2}{2}=\frac{9}{2}=4.5\text{ J}$

8. ④

$F=-\frac{du}{dr}=\frac{k}{r^3}$

$\frac{mv^2}{a}=\frac{k}{a^3} \Rightarrow mv^2=\frac{k}{a^2} \Rightarrow \frac{1}{2}mv^2=\frac{k}{2a^2}$

$\therefore TE=KE+PE=\frac{k}{2a^2}-\frac{k}{2a^2}=0$

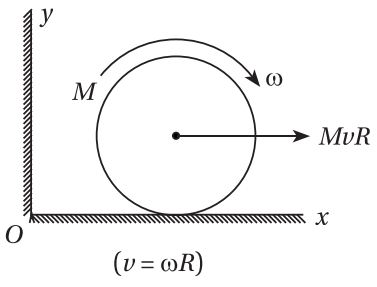
9. ②

$50\times v_1-20\times v_2=0 \qquad v_1$ & $v_2$ : w.r.t. ground

$\therefore\ v_2=2.5\,v_1 \qquad v_1+v_2=0.7 \Rightarrow 3.5v_1=0.7$

$v_1=\frac{7}{35}=\frac{1}{5}=0.2\text{ m/s}$

10. ③




$L_0=MvR+I\omega=MvR+\frac{1}{2}MR^2\omega=\left(\frac{3}{2}\right)MR^2\omega$

11. ③

$\vec{v}=\frac{d\vec{r}}{dt}=\frac{d}{dt}\left(2t\hat{i}-3t^2\hat{j}\right)=2\hat{i}-6t\hat{j}$

$\vec{L}=m(\vec{r}\times\vec{v})=-m\left(6t^2\right)\hat{k} \qquad \vec{L}=\left(-2\times6\times2^2\right)\hat{k}=-48\hat{k}$

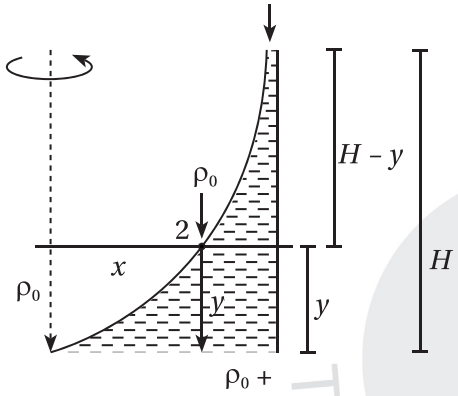
12. ③

$$\frac{mv^2}{r}=\frac{GmM}{r^2}=\frac{Gm\cdot\frac{4}{3}\pi r^3\cdot\rho_0}{r^2}$$

$$\frac{v^2}{r}=\frac{4G\pi r\rho_0}{3} \quad \Rightarrow v\propto r \qquad (\text{when } r<R)$$

$r>R \qquad \frac{mv^2}{r}=\frac{GmM}{r^2} \quad \Rightarrow v\propto\frac{1}{\sqrt{r}}$

13. ④




$$\rho_0=\rho_0+\rho gy-\frac{1}{2}\rho(\omega x)^2$$

$$\therefore y=\frac{\omega^2x^2}{2g}$$

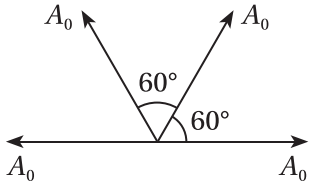
If the fluid is rotating, write $+\left(\frac{1}{2}\rho\omega^2x^2\right)$ while moving away from the axis and $\left(-\frac{\rho\omega^2x^2}{2}\right)$ in moving towards the axis.

$y = 2$ cm

14. ①

$y = kt^2 \quad \Rightarrow \; a_y = 2k = 2\ \text{m/s}^2$ $\qquad \frac{T_1^2}{T_2^2}=\frac{a_y+g}{g}=\frac{6}{5}$

15. ③




$$A^2=A_0^2+A_0^2+2A_0^2\cos60°$$

$A^2=3A_0^2$ $\qquad$ as $I\propto A^2 \quad \therefore I=3I_0 \quad n=3$

16. ①

$I(\pi R^2) = \sigma\left(T^4 - T_0^4\right)4\pi R^2$

(Energy incident per second) = (Energy radiated per second)

$T^4 - T_0^4 = 40\times10^8 \quad \Rightarrow T^4 - 81\times10^4$

$T$ = 330 K

17. ①

$W_{B\to C} = 0 \qquad W_{net} = W_{A\to B} + W_{B\to C} + W_{C\to A}$

$\Rightarrow$ 5 = 10(2 – 1) + 0 + $x$ $\quad \Rightarrow$ $x$ = –5 J

18. ②

On plane surface $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

$\dfrac{1.5}{AI_1} - \dfrac{1}{-mR} = \dfrac{1.5-1}{\alpha} = 0$

$AI_1 = -(1.5mR)$

On curved surface

$\dfrac{1}{\alpha} - \dfrac{1.5}{-(1.5mR+R)} = \dfrac{1-1.5}{-R}$

$\dfrac{3m}{2} = 2 \quad \Rightarrow m = 4/3$

19. ②

$R = \int_a^b dR = \int_a^b \rho\dfrac{dx}{4\pi x^2} = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$

20. ④

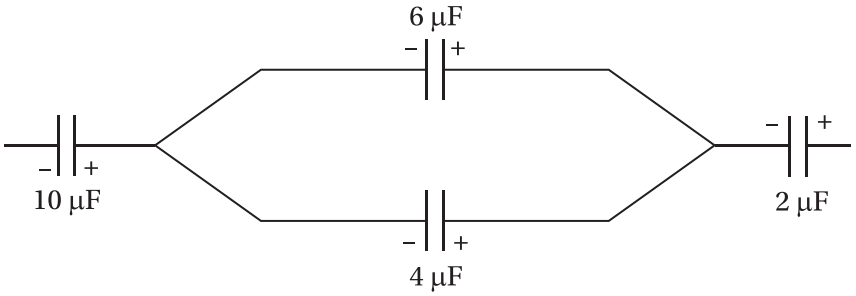
$Q = \int \rho\, dv = \int_0^R \dfrac{A}{r^2}\cdot e^{-2r/a}\left(4\pi r^2\, dr\right) = 2\pi aA\left(1 - e^{-2R/a}\right)$

$R = \dfrac{a}{2}\log\left[\dfrac{1}{1 - \dfrac{Q}{2\pi aA}}\right]$

## SECTION B

**Section B consists of 5 questions of 4 marks each.**

21.




$\frac{q_1}{q_2}=\frac{6v}{4v}=\frac{3}{2}$ $\quad q_2=\frac{2q_1}{3}$

$q_1+\frac{2q_1}{3}=30 \Rightarrow 5q_1=90 \Rightarrow q_1 = 18\ \mu c$ **Ans:** 18

22. $B$ at centre of circle

$=\frac{\mu_0 I}{2R}=\frac{\mu_0 I}{2\cdot\left(\frac{L}{2\pi}\right)}=\pi\left[\frac{\mu_0 I}{L}\right]$

$B$ at centre of square: L = 4a ⇒ a = L/4

$=4\cdot\frac{\mu_0}{4\pi}\cdot\frac{I}{\left(\frac{a}{2}\right)}[\sin 45^\circ+\sin 45^\circ]=\frac{8\sqrt{2}}{\pi}\cdot\left[\frac{\mu_0 I}{L}\right]$

$\frac{B_A}{B_B}=\frac{\pi^2}{8\sqrt{2}}$ $\quad 8=\frac{\pi^2}{\sqrt{2}}\left(\frac{B_B}{B_A}\right)$ **Ans:** 8

23. $I=\frac{e}{R}=\frac{1}{R}\frac{d\phi}{dt} \Rightarrow d\phi = RI\,dt$

$\phi = R\int I\,dt = R\times$ Area under $I-t$ graph = (100)(2.5)=250 wb **Ans:** 250

24. $E_n=-13.6+\frac{12500}{980}=-0.84=-\frac{13.6}{n^2}$

$\Rightarrow n=4$ $\quad r_n=n^2a_0=16a_0 \quad \therefore k=16$

25. $A=A_0\left(\frac{1}{2}\right)^n=\frac{A_0}{16} \Rightarrow n=4$

$t=(n)t_{1/2}=(4)(100)=400\ \mu s$

# CHEMISTRY

## SECTION A

**Section A consists of 20 questions of 4 mark each.**

**26.** ①

F.C. $= V - L - \frac{1}{2}S$

F.C. of B atom $= V - L - \frac{1}{2}S = 3 - 0 - \frac{1}{2} \times 8 = -1$

F.C. of N atom $= 5 - 0 - \frac{1}{2} \times 8 = +1$

$V$ = Valence electron of the central atom; $L$ = Total number of lone pair e's; $S$ = Total number of sharing e's.

**27.** ④

Bond order $\propto$ Bond length;

B.O. of $O_2^+ = 2.5$

B.O. of $O_2 = 2.0$

B.O. of $O_2^- = 1.5$

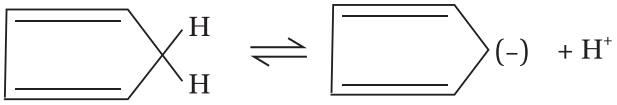
B.O. of $O_2 = 1.0$

$$\text{B.O.} = \frac{N_b - N_a}{2}$$

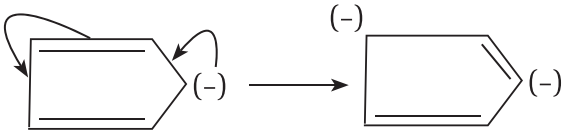
| Totale's | B.O |
|---|---|
| 11 | |
| 12 | |
| 13 ⟶ | 2.5 |
| 14 ⟶ | 3.0 |
| 15 ⟶ | $2.5 \Rightarrow O_2^+$ |
| 16 ⟶ | $2.0 \Rightarrow O2$ |
| 17 ⟶ | $1.5 \Rightarrow O_2^-$ |
| 18 ⟶ | $1.0 \Rightarrow O_2^-$ |

**28.** ④

The acid whose conjugate base is most stable will be strongest acid.

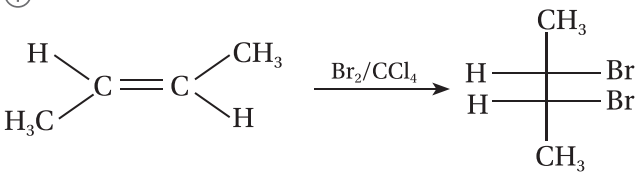



(Aromatic)




The greater the resonance the greater is the stability

**29.** ①




(Trans) (Meso)

30. ④

In S.I. units the P.E. $= \dfrac{-Ze^2}{4\pi\varepsilon_0 r}$

For $Li^{2+}$, $Z = 3$, $\therefore P.E. = \dfrac{-3e^2}{4\pi\varepsilon_0 r}$

31. ①

$r_n = a_0 \times n^2$

$r_4 = a_0(4)^2 = 16a_0$

$mvr = \dfrac{nh}{2\pi}; mv = \dfrac{4h}{2\pi \times 16a_0} = \dfrac{h}{8\pi a_0}$ $\therefore \lambda = \dfrac{h}{mv} = \dfrac{h \times 8\pi a_0}{h} = 8\pi a_0$

32. ①

| Element | % | Atomic weight | R.AM | Simplest ratio |
|---|---|---|---|---|
| A | 25 | 12.5 | $\dfrac{25}{12.5} = 2$ | 1 |
| B | 75 | 37.5 | $\dfrac{75}{37.5} = 2$ | 1 |

$\therefore$ The simplest formula of compound is AB

33. ②

$PCl_5 \rightleftharpoons PCl_3 + Cl_2$

M = $PCl_5$ = 31 + 5 × 35.5 = 31 + 177.5 = 208.5

$\alpha = \dfrac{D-d}{d(n-1)} = \dfrac{104.25-70}{70(2-1)} = 48.9\%$ $D = \dfrac{M}{2} = \dfrac{208.5}{2} = 104.25$

34. ②

NaCN is a salt of strong base and weak acid :

$\therefore pH = 7 + \dfrac{1}{2}pKa + \dfrac{1}{2}\log c$

pKa for HCN = 14 – 4.70 = 9.30

$\therefore pH = 7 + \dfrac{1}{2} \times 9.30 + \dfrac{1}{2} \times \log_{10} 0.5$

$\Rightarrow$ pH = 11.5

35. ③

$\frac{1}{2}H_2 + \frac{1}{2}Cl_2 \rightarrow HCl$

$\Delta H = \varepsilon B.E_{Reactants} - \varepsilon BE_{Products} = \left[\frac{1}{2}B \cdot E(H_2) + \frac{1}{2}B \cdot E(Cl_2)\right] - BE(HCl)$

$= \left[\frac{1}{2} \times 104 + \frac{1}{2} \times 58\right] - 103 = (52 + 29) - 103 = -22 \text{ kcal}$

36. ④

$\Delta G^\circ = -2.303\ RT \log 10\ keq = -nF E^0_{cell}$

If a cell reaction is spontaneous (proceeding in forward side), it means $keq > 1$ and $E^0_{cell}$ = +ve, $\Delta G^\circ$ = –ve

37. ②

| | $Ba(NO_3)_2 \rightleftharpoons$ | $Ba^{++}$ + | $2NO_3^-$ | |
|---|---|---|---|---|
| Initially : | 1 mol | 0 | 0 | Total |
| After Dissociation : | 1 – α | α | 2α | = 1 – α + α + 2α |
| | | | | = 1 + 2α |

Degree of Dissociation : $\alpha = \frac{1-i}{1-x}$; $x$ = number of particles of dissociation

$\Rightarrow \alpha = \frac{1-2.74}{1-3} = \frac{-1.74}{-2} = 0.87 = 87\%$

38. ③

We know, density of solution,

$d = M\left(\frac{1}{m'} + \frac{m}{1000}\right)$

$M$ = Molarity; $m'$ = molality; $m$ = m.w. of NaCl = 23 + 35.5 = 58.5

$\therefore 1.252 = 3\left(\frac{1}{m'} + \frac{58.5}{1000}\right) \Rightarrow 0.417 = \frac{1}{m'} + 0.0585 \Rightarrow m' = 0.279$

39. ④

Reduction = gain of electron

A reducing agent gain electron.

(1) $N^{+5}O_3^- \rightarrow N^{+2}O$

(2) $N^{+4}O_2 \rightarrow N^{+3}O_2^-$

(3) $N^{+6}O_3 \rightarrow N^{+5}H_4^+$

40. ③

$2V^{+2}O + 3Fe^{+3}_2O_3 \rightarrow 6Fe^{+2}O + V^{+5}_2O_5$

$n$-factor = 2(+5) – (2)(+2) = 10 – 4 = +6

41. ③

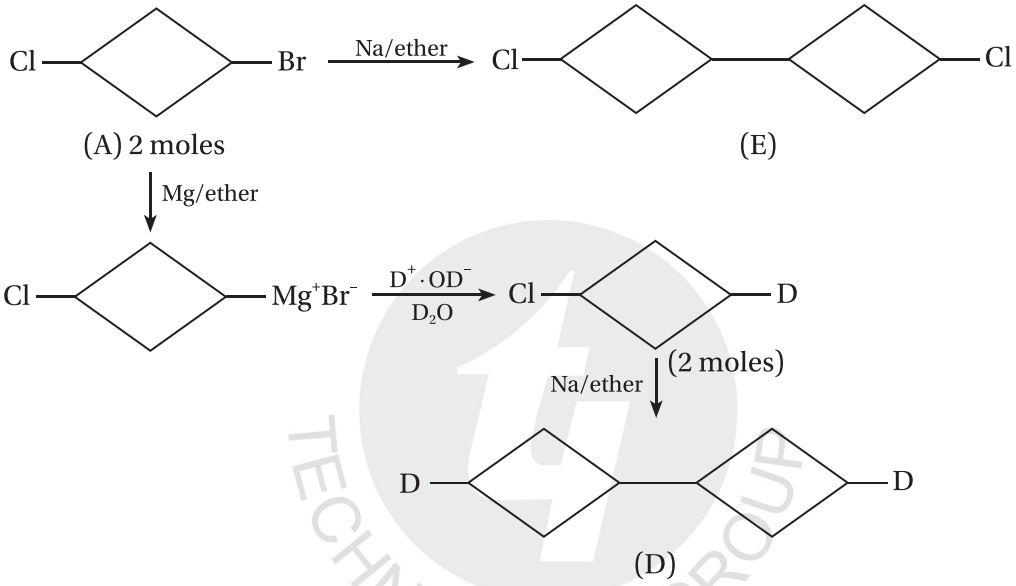
$E^0_{cell} = E^0_{ox} + E^0_{red} = E^0_{Cr/Cr^{3+}} + E^0_{Fe^{++}/Fe} = +0.72 + (-0.42) = +0.30\ V$

The cell reaction is : $2Cr + 3Fe^{2+} \rightarrow 2Cr^{3+} + 3Fe$

$n = 6$

$$E_{cell} = E^0_{cen} - \frac{0.0591}{6}\log\frac{[Cr^{3+}]^2}{[Fe^{2+}]^3} = +0.30\ v - \frac{0.0591}{6}\log\frac{(10^{-1})^2}{(10^{-2})^3} = +0.30\ v - \frac{0.0591}{6}\times 4 = +0.26\ v$$

42. ③




43. ②

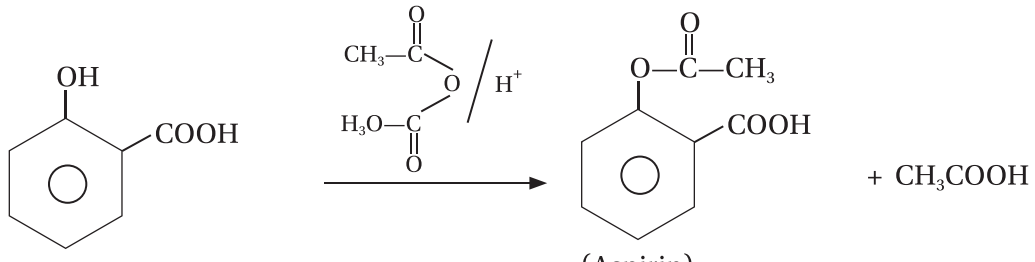
Acidic strength $\propto$ –I effect

As oxygen is more electron with drawing, (II) and (III) show greater –I effect than (I). Thus (I) is least acidic. Out of (II) & (III), II is more acidic than (III) as distance of 'O' increases from —COOH group, acidic strength decrease.

44. ②

The colour of $CrO_3$ ($d^\circ$ configuration) is due to charge transfer from ligand (oxygen) to metal (Cr) and not due to *d–d* transition.

45. ④




(salicylic acid) (Aspirin)

## SECTION B

**Section B consists of 5 questions of 4 marks each.**

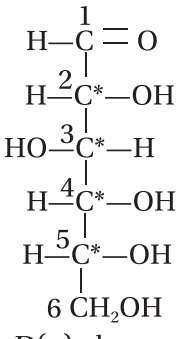
46. Volume of surface = $80 \times 0.0005 = 0.04\ cm^2$

$W_{Ag} = 0.04 \times 10.5 = 0.42\ g$

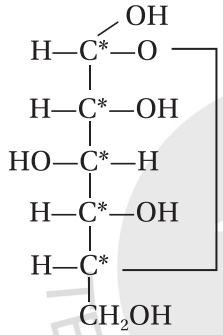
$\therefore W = \frac{i \times t \times E}{96500} \Rightarrow 0.42 = \frac{3 \times t \times 108}{96500} \Rightarrow t = 125.09\ s \approx y^3$

$\therefore\ y = 5$

47. 5;




D(+) glucose



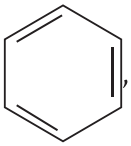

(α–D(+) glucopyranose)

48. 5; $C_4H_{11}N$

Benzene sulphonyl chloride

($C_6H_5SO_2Cl$), also known as Hinsberg's reagent reacts with 1° and 2° ammines to form sulphonamides 1° amine gives alkali soluble ppt.

But 2° amine gives ppt insoluble in alkali. There are 5 1° amine of $C_4H_{11}N$.

49.  , Benzene is the product

($C_6H_6$)

Degree of Unsaturation,

$U = (6+1) - \left(\frac{H-N}{2}\right) = (6+1) - \left(\frac{6-0}{2}\right) = 7 - 3 = 4$

50. 6 ;  gives 6 monochloro derivatives including stereoisomers.

# MATHEMATICS

## SECTION A

**Section A consists of 20 questions of 4 mark each.**

51. ③

$$\frac{2b}{a}=\cos C=\frac{a^2+b^2-c^2}{2ab}$$

$\Rightarrow 3b^2 = a^2 - c^2$

Again $\cos A=\frac{b^2+c^2-a^2}{2bc}=-\frac{b}{c}$

$$\cos A.\cos C=-\frac{b}{c}.\frac{2b}{a}=\frac{2}{3ca}(c^2-a^2)$$

52. ②

At $x = 1$, L.H.D $= -1$

R.H.D $= -1$

At $x = 2$, f is not continuous and hence not differentiable

53. ②

$$f(x)=\frac{x-1}{x+1}$$

$$f^2(x)=\frac{1}{x}$$

$$f^3(x)=\frac{x+1}{1-x}$$

$$f^4(x)=x$$

$$\Rightarrow f^{1999}(x)=f^3(x)=\frac{x+1}{1-x}$$

$$\Rightarrow f^{1999}(2)=-3.$$

54. ②

put $z = x + iy$

$$(x+iy)+c\sqrt{(x+1)^2+y^2}-i=0$$

$$\Rightarrow c\sqrt{(x^2+1)^2}+y^2+x=0,\ y-1=0 \Rightarrow y=1$$

$$x=-\frac{2c^2\pm 2\sqrt{c^4-2c^2(c^2-1)}}{2(c^2-1)}$$

For real $x$ $-\sqrt{2} \le c \le \sqrt{2}$

Since $c \ge 1$ $\therefore 1 \le c \le \sqrt{2}$

If $c = 1 \Rightarrow x = -1$ and $y = 1$

If $c = \sqrt{2} \Rightarrow x = -2, y = 1$

55. ②

Let y = f(x)

$\frac{dy}{dx}+\frac{y}{x}=2 \Rightarrow \text{If}=e^{\int\frac{dx}{x}}=x$

$xf(x)=x^2+c \Rightarrow f(x)=x+\frac{c}{x}=f'(x)=1-\frac{c}{x^2}$

$\lim_{x\to0^+} f'\left(\frac{1}{x}\right)=0$

$\lim_{x\to0^+} x.f\left(\frac{1}{x}\right)=1$

$\lim_{x\to0^+} x^2.f'(x)=-\lambda$

Again, $\lim_{x\to0^+} f(x) = \propto \Rightarrow f(x)$ is not bounded.

56. ①

Equation of tangent at p(h, k)

$(y-k)=\frac{dy}{dx}(x-h)$

$T=\left(\frac{hf'(x)-k}{f'(x)},0\right)$

$PT=\sqrt{\frac{k^2}{f'(x)^2}+kL}=1 \Rightarrow \frac{dy}{dx}=\pm\frac{y}{\sqrt{1-y^2}}$

$\therefore \int\frac{\sqrt{1-y^2}}{y}dy = \pm\int dx.$ put $y=\sin\theta$

$\Rightarrow \log\left|\frac{1-\sqrt{1-y^2}}{y}\right|+\sqrt{1-y^2} = \pm x+c.$

57. ③

$\frac{d}{dx}(e^{f(x)}.\sin f(x)=0$

$\Rightarrow \cos f(x) + \sin f(x) = 0.$ $(\because f'(x) > 0$ and $e^{f(x)} \pm 0)$

$\Rightarrow \cos\left(f(x)-\frac{\pi}{4}\right)=0$

$(f(x))_{min}=\frac{3\pi}{4}=\lambda \quad [\lambda]=2$

58. ②

$^{n}C_{p-1} + {}^{n}C_{p+1} = 2 \times {}^{n}C_{p} \Rightarrow n^2 - n(4p+1) + 4p^2 - 2 = 0$

59. ①

$\lim\limits_{x \to 0} \left( \frac{\sin x}{x} \right)^{\frac{\sin x}{x - \sin x}}$ ($1^{st}$ form)

$= e^{\lim\limits_{x \to 0} \left( \frac{\sin x}{x - \sin x} \right) \left( \frac{\sin x}{x} - 1 \right)} = e^{-1}$

$\therefore \; e^{-1} + e^{-1} = \frac{2}{e}$

60. ③

$T^2 = SS_1$ ; P(d, B) be any point on the locus

$[\beta y - 2a(x + \alpha)]^2 = (B^2 - 4a\alpha)(y^2 - 4ax)$

Again, $\tan 45^\circ = \frac{2\sqrt{h^2 - ab}}{a + b}$

$\Rightarrow (\alpha + 3a)^2 - \beta^2 = 8a^2$

61. ③

$A = \begin{pmatrix} 1 & 5 & 25 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{pmatrix}$

$A^2 = \begin{pmatrix} 1 & 10 & 25(1+2) \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix}$

$A^3 = \begin{pmatrix} 1 & 15 & 25(1+2+3) \\ 0 & 1 & 10 \\ 0 & 0 & 1 \end{pmatrix}$

$\vdots$

By Induction, $A^{50} = \begin{bmatrix} 1 & 250 & \frac{25 \times 50 \times 51}{2} \\ 0 & 1 & 250 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow b_{23} = 250, \; b_{12} = 250$

$\Rightarrow \frac{2b_{13}}{b_{12}} + \frac{2b_{23}}{b_{12}} = 257$

62. ③

$y^2 = 8ax \rightarrow (2a, 0)$ focus

$\rightarrow (10a, 0)$ point

Normal: $y = mx - 4am - 2am^3$ passes through $(10a, 0)$

$\Rightarrow m^2 = 3 \quad (m \pm 0) \Rightarrow m = \pm\sqrt{3} \Rightarrow \tan\theta = \frac{\pi}{3}.$

63. ②

Let $P = (a\cos\theta, b\sin\theta) \quad Q = (a\cos\theta, a\sin\theta)$

$$\therefore R = \left(\frac{ra\cos\theta + 8a\cos\theta}{r+s}, \frac{ra\sin\theta + rb\sin\theta}{r+s}\right) = (h, k)$$

$$\therefore \left.\begin{array}{l} \frac{n}{a} = \cos\theta \\ \frac{K(r+s)}{ra+Sb} = \sin\theta \end{array}\right\} \quad \frac{n^2}{a^2} + \frac{K^2(r+S)^2}{(ra+Sb)^2} = 1$$

64. ①

We have $|az_1 + bz_2|^2 + |bz_1 - az_2|^2 = |az_1|^2 + |bz_2|^2 + 2Re(az_1\overline{bz_2}) + |bz_1|^2 + |az_2|^2 - 2Re(bz_1.\overline{az_2})$

$= a^2|z_1|^2 + b^2|z_2|^2 + 2abRe(z_1\bar{z}_2) + b^2|z_1|^2 + a^2|z_2|^2 - 2abRe(z_1\bar{z}_2)$

($\because$ a and b are real numbers)

$= (a^2 + b^2)(|z_1|^2 + |z_2|^2)$

65. ③

$$\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = K \Rightarrow a = 7K,\ b = 6K,\ C = 5K$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{5}$$

$$\cos B = \frac{19}{35}$$

$$\cos C = \frac{5}{7}$$

$\therefore \cos A : \cos B : \cos C = 7 : 19 : 25$

66. ②

$T = S_1 \Rightarrow hx + ky = h^2 + k^2 \quad \ldots (1)$

COC at $(3\sec\theta, 2\tan\theta) \Rightarrow 3\sec\theta x + 2\tan\theta y = 9 \quad \ldots (2)$

From (1) & (2): $\sec\theta = \frac{3h}{h^2+k^2}, \tan\theta = \frac{9k}{2(h^2+k^2)}$

$\Rightarrow$ locus is: $\frac{9x^2}{(h^2+y^2)^2} - \frac{81y^2}{4(x^2+y^2)^2} = 1$

67. ④

00, 01, 02, 03, ... 99

A = {29, 36, 63, 92}

$P(A) = \frac{4}{100} = 0.04$

$P(\bar{A}) = 0.96$

Required probability $= {}^8C_3 \cdot \left(\frac{4}{100}\right)^3 \left(\frac{96}{100}\right)^5 = {}^8C_3\,(0.04)^3(0.96)^5$

68. ①

D = – $(1 - a)^2$ for no solution a = 1

x + y + z = 1

x + y + z = 1

x + by + z = 0

b = 1 for no solution

69. ②

$[f(x)] = \begin{cases} [\frac{1}{2}x - 1] = -1 & ; \quad 0 < x < 2 \\ 0 & , \quad 2 \le x \le \pi \end{cases}$

$\therefore$ tan [f(x)] = tan(–1) = – tan 1, $0 < x < 2$

$= 0, \quad 2 < x < \pi$

$\Rightarrow$ [f(x)] is discontinuous at x = 2

Let, $f^{-1}(x) = y \Rightarrow x = f(y)$

$\Rightarrow f^{-1}(x) = 2(x + 1)$ continuous on $[0, \pi]$

$\frac{1}{f(x)}$ is not defined at x = 2

70. ①

B is equivalence relation

For A: x = x + 1

$\Rightarrow$ 0 = 1 impossible

$\therefore$ A is not equivalence relation

## SECTION B

**Section B consists of 5 questions of 4 marks each.**

71. (4)

$f(n) = 2^n$

$f(a + 1) + (a + 2) + ... + f(a + x) = 32.(2^n - 1)$

$\Rightarrow f(a)\,[f(1) + ... + f(x)] = 32.(2^n - 1)$

$\Rightarrow f(a).\frac{2.2^n - 1}{2 - 1} = 32(2^n - 1) \Rightarrow f(a) = 16 = 2^4 \quad \therefore a = 4$

72. (5)

$\frac{x^2}{6}+\frac{y^2}{2}=1$

director circle : $x^2 + y^2 = 8$

Equation of C.O.C of ellipse w.r.t $(\sqrt{8}\cos\theta, \sqrt{8}\sin\theta)$

$(\sqrt{8}\cos\theta)x+(3\sqrt{8}\sin\theta)y-6=0$. Suppose it touches the circle $x^2 + y^2 = r^2$

$$r=\left|\frac{-6}{\sqrt{8\cos^2\theta+72\sin^2\theta}}\right| \Rightarrow r^2=\frac{36}{8+64\sin^2\theta} \Rightarrow \begin{cases} r^2_{max}=\frac{9}{2} \\ r^2_{min}=\frac{1}{2} \end{cases}$$

Max area + Min area $=\pi\left(\frac{9}{2}\right)+n\left(\frac{1}{2}\right)=5\pi$

73. (816)

$tn = n \times {}^nP_n = n \cdot n! = (n+1)! - n!$

$\sum_{n=1}^{16} t_n = 17! - 1 = {}^qP_r - s = {}^{17}P_{16} - s \quad \Rightarrow q = 17, r = 16, s = 1 \quad (q \neq r)$

${}^{q+s}C_{r-s} = {}^{18}C_{15} = 816$

74. (64)

Let $X = x - \frac{5}{4}$

$Y = y - \frac{4}{3}$

$Z = z + 5$

$4|X| + 3|Y| + |Z| = 4$

The planes: $\left.\begin{aligned} 4X+3Y+Z &= \pm 4 \\ -4X+3Y+Z &= \pm 4 \\ 4X-3Y+Z &= \pm 4 \\ 4X+3Y-Z &= \pm 4 \end{aligned}\right\}$ form octahedron

The plane $4X + 3Y + Z = 4$ forms a tetrahedron with coordinates plane in the positive octant of volume

$\frac{1}{6} \cdot 1 \cdot \frac{4}{3} \cdot 4 = \frac{8}{9}$

$\therefore$ Total volume $=8\left(\frac{8}{9}\right)=\frac{64}{9}.$

75. (1)

$2x = \sin\left(\frac{\pi}{3} - \cos^{-1}x\right) = \frac{\sqrt{3}}{2}x - \frac{\sqrt{1-x^2}}{2}$

$\Rightarrow \quad 4x = \sqrt{3}x - \sqrt{1-x^2}$

$\Rightarrow \quad x^2 = \frac{1}{20-8\sqrt{3}}; \quad x = -\frac{1}{\sqrt{20-8\sqrt{3}}} \quad (x < 0)$